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时滞中立型 Lurie 系统的绝对稳定性分析

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摘 要: 研究了具有时滞的中立型 Lurie 控制系统的绝对稳定性问题. 通过把离散时滞区间分割成多个部分和构造适当的 Lyapunov-Krasovskii 泛函,分别给出系统在无限扇形角和有限扇形角内绝对稳定的时滞相关充分条件. 所给的判定条件既与离散时滞相关又与中立型时滞相关,而且是线性矩阵不等式形式,可以方便地运用 Matlab 工具箱求解. 最后数值例子说明本方法的有效性和可行性.

关键词: Lurie 控制系统;时滞相关稳定;中立型时滞;线性矩阵不等式

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0 引言

许多的工程系统中都会存在时滞现象,时滞的存在往往是系统不稳定和系统性能变差的根源,因此,时滞系统的稳定性研究在理论上和实际中都有重要意义. 中立型系统在生活中广泛存在,如重复控制系统,分布式网络包括无损传输线、种群生态学、热量交换等系统[1-2],所以中立型时滞系统的稳定性分析得到越来越多国内外学者的重视.

众所周知,Lurie 型控制系统是一类非常重要的非线性系统,许多实际问题都可以转化成 Lurie 型控制系统. 自从绝对稳定性的概念提出以来,Lurie 系统的稳定性问题就受到了广泛的关注,也取得了许多有价值的研究成果[3-10]. 其中,文献[3-5]考虑了一类 Lurie 系统的稳定性问题,得到了一些稳定性充分条件,但是这些条件是时滞无关的. 一般认为时滞无关条件具有保守性,特别是在时滞较小时. 文献[6-7]基于确定模型变换和矩阵分解方法,结合积分不等式技术得到了 Lurie 系统绝对稳定的时滞相关条件. 然而确定模型变换方法会引入一些新的动态因素,导致所得结果具有保守性. 到目前为止,很少有文献研究中立型 Lurie 系统的稳定性问题[8-10],而且所得结论几乎都只与离散时滞相关而与中立型时滞无关. 王岩青等[10]通过构造特殊的 Lyapunov 泛函,利用矩阵不等式技术得到时滞相关充分条件,但所研究的中立型 Lurie 系统的离散时滞和中立型时滞相等,所以该文结果具有一定的局限性.

笔者在文献[10]模型的基础上研究了一类离散时滞和中立型时滞不相等的中立型 Lurie 系统的稳定性问题,介绍了一种研究该系统的有效方法. 通过把离散时滞区间分割成多个部分,使每个部分都有一个不同的 Lyapunov 泛函,利用 LMI 技术分别得到了该系统在无限扇形角和有限扇形角内绝对稳定的充分条件,这些条件既与离散时滞相关又与中立时滞相关,所以结果具有更低的保守性.

1 预备知识与系统描述

考虑如下具有不相等中立时滞和状态时滞[11](通常称为离散时滞)的中立型 Lurie 直接系统

$$\begin{cases} \dot{x}(t) - \boldsymbol{B}_d\dot{x}(t-d) = \boldsymbol{A}x(t) + \boldsymbol{B}x(t-h) + \boldsymbol{D}f(\sigma(t)); \\ \sigma(t) = \boldsymbol{C}^{\mathrm{T}}x(t); \\ x(t) = \boldsymbol{f}(t), \forall t \in [-\max\{h,d\},0]. \end{cases} \tag{1}$$

式中: $\boldsymbol{x}(t) \in R^n$ 为状态变量; $\boldsymbol{A},\boldsymbol{B},\boldsymbol{B}_d \in R^{n\times n}$; $\boldsymbol{C}, \boldsymbol{D} \in R^{n\times m}$; $\tau,h \geqslant 0$ 为常数时滞; 向量 $\boldsymbol{\sigma}(t) = (\sigma_1(t),\sigma_2(t),\cdots,\sigma_m(t))^{\mathrm{T}}$, 非线性函数 $\boldsymbol{f}(\sigma(t)) = (f_1(\sigma_1(t)),f_2(\sigma_2(t))\cdots,f_m(\sigma_m(t)))^{\mathrm{T}}$, $f_j(\cdot)$ 满足无限扇形条件:

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$$f_j(\cdot) \in k_j[0,\infty) = \{f_j(\cdot)|f_j(0) = 0, \sigma_j f_j(\sigma_j) > 0, \sigma_j \neq 0\}, \quad (2)$$

或有限扇形条件：

$$f_j(\cdot) \in k_j[0,k_j] = \{f_j(\cdot)|f_j(0) = 0, 0 < \sigma_j f_j(\sigma_j) \leq k_j\sigma_j^2, \sigma_j \neq 0\}. \quad (3)$$

为了获得系统稳定的最大时滞上限，把离散时滞 h 分为 N(正整数)部分，每一部分的长度为 $\tau = \frac{h}{N}$.

为了文后讨论的方便，我们引入如下的定义和引理：

首先定义算子 Δ: $C([-d,0],R^n) \to R^n$, $\Delta(t) = x(t) - \boldsymbol{B}_d\boldsymbol{x}(t-d)$.

定义 1 如果齐次微分方程的 $\Delta(t) = 0, t \geq 0, x_0 = \psi \in \{\phi \in C[-d,0]: \Delta\phi = 0\}$ 零解一致渐近稳定，那么算子 Δ 是稳定的.

引理 1[12] 设常数 $\gamma > 0$，向量值函数 $\omega(s)$: $[0,\gamma] \to R^n$ 可积，则对任意对称正定矩阵 $\boldsymbol{R} \in R^{n\times n}$，不等式成立：$\left(\int_0^\gamma \boldsymbol{\omega}(s)\mathrm{d}s\right)^{\mathrm{T}} \boldsymbol{R}\left(\int_0^\gamma \omega(s)\mathrm{d}s\right) \leq \gamma\int_0^\gamma \boldsymbol{\omega}^{\mathrm{T}}(s)\boldsymbol{R}\boldsymbol{\omega}(s)\mathrm{d}s$.

引理 2 Schur 补引理：对给定的对称矩阵 $\boldsymbol{S} = \boldsymbol{S}^{\mathrm{T}} = \begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} \\ * & \boldsymbol{S}_{22} \end{bmatrix}$，其中 $\boldsymbol{S}_{11} \in \mathrm{R}^{r\times r}$. 以下 3 个条件是等价的：(1) $\boldsymbol{S} < 0$;

(2) $\boldsymbol{S}_{11} < 0, \boldsymbol{S}_{22} - \boldsymbol{S}_{12}^{\mathrm{T}}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12} < 0$;

(3) $\boldsymbol{S}_{22} < 0, \boldsymbol{S}_{11} - \boldsymbol{S}_{12}\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{12}^{\mathrm{T}} < 0$.

2 主要结果

首先考虑非线性函数 $f_j(\cdot), j = 1,2,\cdots,m$ 在满足无限扇形条件的情况下，系统的绝对稳定性.

定理 1 给定标量 $d > 0$ 和 $\tau > 0$，如果算子 Δ 是稳定的，并且存在对称正定矩阵 $\boldsymbol{P}, \boldsymbol{R}_1, \boldsymbol{W}_j, \boldsymbol{Q}_i, \boldsymbol{Z}_i (i = 1,2,\cdots,N; j = 1,2)$，常数 $\alpha > 0$ 和对角矩阵 $\boldsymbol{R} = diag\{r_1, r_2, \cdots, r_m\} \geq 0$ 使得如下 LMI 成立：

$$\boldsymbol{\Theta} = \begin{bmatrix} \boldsymbol{\Theta}_1 & \boldsymbol{\Theta}_2 & \boldsymbol{\Gamma}^{\mathrm{T}}\boldsymbol{F} \\ \boldsymbol{\Theta}_2^{\mathrm{T}} & \boldsymbol{\Theta}_3 & 0 \\ \boldsymbol{F}^{\mathrm{T}}\boldsymbol{\Gamma} & 0 & -\boldsymbol{F} \end{bmatrix} < 0, \quad (4)$$

则具有多非线性执行机构的系统(1)在无限扇形条件(2)下是绝对稳定的. 其中，

$$\boldsymbol{\Theta}_1 = \begin{bmatrix} \boldsymbol{\Theta}_{11} & \boldsymbol{PB} & \boldsymbol{W}_2 - \boldsymbol{A}^T\boldsymbol{P}\boldsymbol{B}_d & 0 & \boldsymbol{\Theta}_{15} \\ * & -\boldsymbol{Q}_N - \boldsymbol{Z}_N & 0 & 0 & \boldsymbol{B}^{\mathrm{T}}\boldsymbol{CR} \\ * & * & \boldsymbol{\Theta}_{33} & 0 & -\boldsymbol{B}_d^{\mathrm{T}}\boldsymbol{PD} \\ * & * & * & -\boldsymbol{W}_1 & \boldsymbol{B}_d^{\mathrm{T}}\boldsymbol{CR} \\ * & * & * & * & \boldsymbol{\Theta}_{55} \end{bmatrix};$$

$$\boldsymbol{\Theta}_2 = \begin{bmatrix} \boldsymbol{Z}_1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & \boldsymbol{Z}_n \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix};$$

$$\boldsymbol{\Theta}_3 = \begin{bmatrix} \boldsymbol{M}_1 & \boldsymbol{Z}_2 & & & \\ \boldsymbol{Z}_2 & \boldsymbol{M}_2 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \boldsymbol{M}_{N-2} & \boldsymbol{Z}_{N-1} \\ & & & \boldsymbol{Z}_{N-1} & \boldsymbol{M}_{N-1} \end{bmatrix},$$

$$\boldsymbol{\Gamma} = [\boldsymbol{A}\ \boldsymbol{B}\ 0\ \boldsymbol{B}_d\ \boldsymbol{D}], \boldsymbol{F} = \boldsymbol{W}_1 + d^2\boldsymbol{W}_2 + \tau^2\sum_{i=1}^N \boldsymbol{Z}_i;$$

$$\boldsymbol{\Theta}_{11} = \boldsymbol{PA} + \boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{R}_1 - \boldsymbol{W}_2 + \boldsymbol{Q}_1 - \boldsymbol{Z}_1;$$

$$\boldsymbol{\Theta}_{33} = -\boldsymbol{R}_1 - \boldsymbol{W}_2 - \boldsymbol{B}^T\boldsymbol{P}\boldsymbol{B}_d, \boldsymbol{\Theta}_{15} = \boldsymbol{PD} + \alpha\boldsymbol{C} + \boldsymbol{A}^T\boldsymbol{CR};$$

$$\boldsymbol{\Theta}_{55} = \boldsymbol{D}^T\boldsymbol{CR} + \boldsymbol{R}\boldsymbol{C}^T\boldsymbol{D}, \boldsymbol{M}_i = -\boldsymbol{Q}_i + \boldsymbol{Q}_{i+1} - \boldsymbol{Z}_i - \boldsymbol{Z}_{i+1}.$$

证明：选取如下形式的 Lyapunov 泛函

$$V(t) = \boldsymbol{V}_1(t) + \boldsymbol{V}_2(t) + \boldsymbol{V}_3(t) + \boldsymbol{V}_4(t) + \boldsymbol{V}_5(t); \quad (5)$$

$$\boldsymbol{V}_1(t) = \Delta^{\mathrm{T}}(t)\boldsymbol{P}\Delta(t), \quad (6)$$

$$\boldsymbol{V}_2(t) = \int_{t-d}^t \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{R}_1 x(s)\mathrm{d}s, \quad (7)$$

$$\boldsymbol{V}_3(t) = \int_{t-d}^t \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{W}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s + \sum_{i=1}^N \int_{t-i\tau}^{t-(i-1)\tau} \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_i\boldsymbol{x}(s)\mathrm{d}s, \quad (8)$$

$$\boldsymbol{V}_4(t) = \int_{-d}^0\int_{t+\theta}^t \mathrm{d}\dot{x}^{\mathrm{T}}(s)\boldsymbol{W}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta + \sum_{i=1}^N \int_{-i\tau}^{-(i-1)\tau}\int_{t+\theta}^t \tau\dot{\boldsymbol{x}}^T(s)Z_i\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta, \quad (9)$$

$$\boldsymbol{V}_5(t) = 2\sum_{i=1}^m r_i\int_0^{\sigma_i} f_i(s)\mathrm{d}s. \quad (10)$$

其中，$\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} > 0; \boldsymbol{R}_1 = \boldsymbol{R}_1^{\mathrm{T}} > 0; \boldsymbol{W}_j = \boldsymbol{W}_j^{\mathrm{T}} > 0; \boldsymbol{Q}_i = \boldsymbol{Q}_i^{\mathrm{T}} > 0; \boldsymbol{Z}_i = \boldsymbol{Z}_i^{\mathrm{T}} > 0 (i = 1,2,\cdots,N; j = 1,2)$ 和 $\boldsymbol{R} = diag\{r_1, r_2, \cdots, r_m\} \geq 0$ 对角矩阵是待定矩阵. 计算上述 Lyapunov 泛函沿系统(1)的解的导数有

$$\dot{\boldsymbol{V}}_1(t) = 2\Delta^{\mathrm{T}}(t)\boldsymbol{P}\dot{\Delta}(t) = 2(^*(t) - \boldsymbol{B}_d\boldsymbol{x}(t-d))^{\mathrm{T}}.$$

$$P(Ax(t)+Bx(t-h)+Df(\sigma(t)))=$$
$$x^{\mathrm T}(t)(PA+A^{\mathrm T}P)x(t)+2x^{\mathrm T}(t)PBx(t-h)+$$
$$2x^{\mathrm T}(t)PDf(\sigma(t))-2x^{\mathrm T}(t)A^{\mathrm T}PB_d\dot x(t-d)-$$
$$2x^{\mathrm T}(t-h)B^{\mathrm T}PB_d\dot x(t-d)-$$
$$2f^{\mathrm T}(\sigma(t))D^{\mathrm T}PB_d\dot x(t-d). \qquad (11)$$

$$\dot V_2(t)=x^{\mathrm T}(t)R_1x(t)-x^{\mathrm T}(t-d)R_1x(t-d). \qquad (12)$$

$$\dot V_3(t)=\dot x^{\mathrm T}(t)W_1\dot x(t)-\dot x^{\mathrm T}(t-d)W_1\dot x(t-d)+\sum_{i=1}^{N}[x^{\mathrm T}(t-(i-1)\tau)Q_ix(t-(i-1)\tau)-x^{\mathrm T}(t-i\tau)Q_ix(t-i\tau)]. \qquad (13)$$

无限扇形条件(2)等价于 $2\alpha x^{\mathrm T}(t)Cf(\sigma(t))\geqslant 0$,其中 α 为大于 0 的常数.

$$\dot V_5(t)=2f^{\mathrm T}(\sigma(t))RC^{\mathrm T}\dot x(t)\leqslant$$
$$2f^{\mathrm T}(\sigma(t))RC^{\mathrm T}[Ax(t)+Bx(t-h)+B_d\dot x(t-d)+Df(\sigma(t))]+2\alpha x^{\mathrm T}(t)Cf(\sigma(t))=$$
$$2f^{\mathrm T}(\sigma(t))(RC^{\mathrm T}A+\alpha C^{\mathrm T})x(t)+$$
$$2f^{\mathrm T}(\sigma(t))RC^{\mathrm T}Bx(t-h)+$$
$$2f^{\mathrm T}(\sigma(t))RC^{\mathrm T}B_d\dot x(t-d)+$$
$$2f^{\mathrm T}(\sigma(t))RC^{\mathrm T}Df(\sigma(t)). \qquad (14)$$

$$\dot V_4(t)=d^2\dot x^{\mathrm T}(t)W_2\dot x(t)-d\int_{t-d}^{t}\dot x^{\mathrm T}(s)W_2\dot x(s)\mathrm ds+\sum_{i=1}^{N}[\tau^2\dot x^{\mathrm T}(t)Z_i\dot x(t)-\tau\int_{t-\tau i}^{t-(i-1)\tau}\dot x^{\mathrm T}(s)Z_i\dot x(s)\mathrm ds]$$

利用引理 1 可得:

$$-d\int_{t-d}^{t}\dot x^{\mathrm T}(s)W_2\dot x(s)\mathrm ds\leqslant-(\int_{t-d}^{t}\dot x(s)ds)TW_2(\int_{t-d}^{t}\dot x(s)\mathrm ds)=$$
$$-x^{\mathrm T}(t)W_2x(t)-x^{\mathrm T}(t-d)W_2x(t-d)+x^{\mathrm T}(t)W_2x(t-d). \qquad (15)$$

$$-\sum_{i=1}^{N}\tau\int_{t-\tau i}^{t-(i-1)\tau}\dot x^{\mathrm T}(s)Z_i\dot x(s)\mathrm ds\leqslant$$
$$-(\int_{t-i\tau}^{t-(i-1)\tau}\dot x(s)\mathrm ds)^{\mathrm T}Z_i(\int_{t-i\tau}^{t-(i-1)\tau}\dot x(s)\mathrm ds)$$
$$=-\sum_{i=1}^{N}(x(t-(i-1)\tau)-x(t-i\tau))^{\mathrm T}Z_i(x(t-(i-1)\tau)-x(t-i\tau)). \qquad (16)$$

所以

$$\dot V_4(t)\leqslant\dot x^{\mathrm T}(t)(d^2W_2+\tau^2\sum_{i=1}^{N}Z_i)\dot x(t)-x^{\mathrm T}(t)W_2x(t)$$
$$-x^{\mathrm T}(t-d)W_2x(t-d)+x^{\mathrm T}(t)W_2x(t-d)$$
$$-\sum_{i=1}^{N}(x(t-(i-1)\tau)-x(t-i\tau))^{\mathrm T}Z_i(x(t-(i-1)\tau)-x(t-i\tau)). \qquad (17)$$

由上述(11)-(17)式可得:

$$\dot V(t)=\dot V_1(t)+\dot V_2(t)+\dot V_3(t)+\dot V_4(t)+\dot V_5(t)$$
$$\leqslant\xi^{\mathrm T}(t)\begin{bmatrix}\Theta_1+\Gamma^{\mathrm T}F\Gamma & \Theta_2\\ \Theta_2^{\mathrm T} & \Theta_3\end{bmatrix}\xi(t)$$

其中,$\xi(t)=[x^{\mathrm T}(t),x^{\mathrm T}(t-h),x^{\mathrm T}(t-d),\dot x^{\mathrm T}(t-d),f(\sigma(t)),x^{\mathrm T}(t-\tau),x^{\mathrm T}(t-2\tau),\cdots,x^{\mathrm T}(t-(N-1)\tau)]^{\mathrm T}$,$\Theta_1$,$\Theta_2$,$\Theta_3$,$F$,$\Gamma$ 定义在定理 1 中. 这样当 $\begin{bmatrix}\Theta_1+\Gamma^{T}F\Gamma & \Theta_2\\ \Theta_2^{\mathrm T} & \Theta_3\end{bmatrix}<0$ 时,$\dot V(t)\leqslant-\varepsilon\|x(t)\|^2$ 对充分小的 $\varepsilon>0$ 成立. 利用 Schur 补,$\begin{bmatrix}\Theta_1+\Gamma^{T}F\Gamma & \Theta_2\\ \Theta_2^{\mathrm T} & \Theta_3\end{bmatrix}<0$ 等价于不等式(4)成立. 注意到算子 Δ 是稳定的,那么当不等式(4)成立时系统(1)在定理 1 的条件下是绝对稳定的.

考虑非线性函数 $f_j(\cdot)$,$j=1,2,\cdots,m$ 在满足有限扇形条件的情况下即 $f_j(\cdot)\in[0,K]$,$K=\mathrm{diag}\{k_1,k_2,\cdots,k_m\}$,系统的绝对稳定性.

定理 2　给定标量 $d>0$ 和 $\tau>0$,如果算子 Δ 是稳定的,并且存在对称正定矩阵 $P,R_1,W_j,Q_i,Z_i(i=1,2,\cdots,N;j=1,2)$,常数 $\alpha>0$ 和对称矩阵 $R=\mathrm{diag}\{r_1,r_2,\cdots,r_m\}\geqslant 0$ 使得如下 LMI 成立:

$$\bar\Theta=\begin{bmatrix}\bar\Theta_1 & \Theta_2 & \Gamma^{T}F\\ \Theta_2^{\mathrm T} & \Theta_3 & 0\\ F^{\mathrm T}\Gamma & 0 & -F\end{bmatrix}<0, \qquad (18)$$

则具有多非线性执行机构的系统(1)在有限扇形条件(3)下是绝对稳定的. 其中,

$$\bar\Theta_1=\begin{bmatrix}\Theta_{11} & PB & W_2-A^{\mathrm T}PB_d & 0 & \bar\Theta_{15}\\ * & -Q_N-Z_N & 0 & 0 & B^{\mathrm T}CR\\ * & * & \Theta_{33} & 0 & -B_d^{\mathrm T}PD\\ * & * & * & -W_1 & B_d^{\mathrm T}CR\\ * & * & * & * & \bar\Theta_{55}\end{bmatrix};$$

$\Gamma=[A\ B\ 0\ B_d\ D]$,$F=W_1+d^2W_2+\tau^2\sum_{i=1}^{N}Z_i$;

$\Theta_{11}=PA+A^{\mathrm T}P+R_1-W_2+Q_1-Z_1$;

$\Theta_{33}=-R_1-W_2-B^{\mathrm T}PB_d$,$\bar\Theta_{15}=PD+\alpha CK+A^{\mathrm T}CR$,$\bar\Theta_{55}=D^{\mathrm T}CR+RC^{\mathrm T}D-2\alpha I$;

$$\Theta_2=\begin{bmatrix}Z_1 & 0 & \cdots & 0 & 0\\ 0 & 0 & \cdots & 0 & Z_n\\ 0 & 0 & \cdots & 0 & 0\\ 0 & 0 & \cdots & 0 & 0\\ 0 & 0 & \cdots & 0 & 0\end{bmatrix};$$

$$\boldsymbol{\Theta}_3=\begin{bmatrix} \boldsymbol{M}_1 & \boldsymbol{Z}_2 & & & \\ \boldsymbol{Z}_2 & \boldsymbol{M}_2 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \boldsymbol{M}_{N-2} & \boldsymbol{Z}_{N-1} \\ & & & \boldsymbol{Z}_{N-1} & \boldsymbol{M}_{N-1} \end{bmatrix};$$

$$\boldsymbol{M}_i=-\boldsymbol{Q}_i+\boldsymbol{Q}_{i+1}-\boldsymbol{Z}_i-\boldsymbol{Z}_{i+1}.$$

证明:有限扇形条件(3)等价于

$$f_j(\sigma_j(t))[k_jc_jx_j(t)-f_j(\sigma_j(t))]\geqslant 0,\quad j=1,2,\cdots,m. \tag{19}$$

用(19)式替换 $\dot{V}_5(t)$ 中的 $2\alpha\,\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{C}\boldsymbol{f}(\sigma(t))$ 得到

$$\begin{aligned}\dot{V}_5(t)&=2\boldsymbol{f}^{\mathrm{T}}(\sigma(t))\boldsymbol{R}\,\boldsymbol{C}^{\mathrm{T}}\dot{\boldsymbol{x}}(t)\leqslant\\ &2\boldsymbol{f}^{\mathrm{T}}(\sigma(t))\boldsymbol{R}\boldsymbol{C}^{\mathrm{T}}[\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{x}(t-h)+\\ &\boldsymbol{B}_d\dot{\boldsymbol{x}}(t-d)+\boldsymbol{D}\boldsymbol{f}(\sigma(t))]+\\ &2\alpha\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{\sigma}(t))[\boldsymbol{K}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{x}(t)-\boldsymbol{f}(\sigma(t))]=\\ &2\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{\sigma}(t))(\boldsymbol{R}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{A}+\alpha\boldsymbol{K}\boldsymbol{C}^{\mathrm{T}})\boldsymbol{x}(t)+\\ &2\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{\sigma}(t))\boldsymbol{R}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{x}(t-h)+2\boldsymbol{f}^{\mathrm{T}}(\sigma(t))\boldsymbol{R}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{B}_d\dot{\boldsymbol{x}}(t-d)+\\ &\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{\sigma}(t))(\boldsymbol{R}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{D}+\boldsymbol{D}^{\mathrm{T}}\boldsymbol{C}\boldsymbol{R}-2\alpha\boldsymbol{I})\boldsymbol{f}(\boldsymbol{\sigma}(t)).\end{aligned} \tag{20}$$

利用式(11)、(12)、(13)、(17)和式(20)可以得到:$\dot{V}(t)\leqslant\boldsymbol{\xi}^{\mathrm{T}}(t)\begin{bmatrix}\overline{\boldsymbol{\Theta}}_1+\boldsymbol{\Gamma}^{\mathrm{T}}\boldsymbol{F}\boldsymbol{\Gamma} & \boldsymbol{\Theta}_2\\ \boldsymbol{\Theta}_2^{\mathrm{T}} & \boldsymbol{\Theta}_3\end{bmatrix}\boldsymbol{\xi}(t).$

类似于定理1的证明过程,可以得到当不等式(18)成立时系统(1)在定理2的条件下是绝对稳定的.

3 数值例子

考虑系统(1)的绝对稳定性,其中各系数矩阵为:

$$\boldsymbol{A}=\begin{pmatrix}-0.9 & 0.2\\ 0.1 & -0.9\end{pmatrix};\ \boldsymbol{B}=\begin{pmatrix}-1.1 & -0.2\\ -0.1 & -1.1\end{pmatrix};$$

$$\boldsymbol{D}=\begin{pmatrix}-0.2 & 0.1\\ -0.45 & -0.3\end{pmatrix};\ \boldsymbol{B}_d=\begin{pmatrix}-0.2 & 0\\ 0.2 & -0.1\end{pmatrix};$$

$$\boldsymbol{C}=\begin{pmatrix}0.3 & -0.2\\ 0.3 & 0.1\end{pmatrix}.$$

利用Matlab软件中的线性矩阵不等式工具箱求解定理1中的LMI(4)可得,当 $N=3$ 时,最大时滞上界为 $d=h=1.990\,5$,所以当 $d=h\leqslant 1.990\,5$ 时系统(1)是绝对稳定的.

4 结论

笔者研究了一类离散时滞和中立型时滞不相等的中立型Lurie控制系统的绝对稳定性问题.基于Lyapunov泛函方法和线性矩阵不等式技术,通过离散时滞区间分割方法,得到了保证系统绝对稳定的保守性更低的稳定性判据,所得结果既与离散时滞相关又与中立型时滞相关.最后通过适当的算例加以验证,说明本文方法是有效的且易于验证.此外,笔者提出的方法能够很容易地扩展应用到时滞中立型系统的稳定性或指数稳定性问题中.

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Absolute Stability Analysis of Neutral Lurie Systems with Time Delays

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Abstract: The absolute stability problem for neutral Lurie control systems with time delays is studied. By dividing the discrete delay interval into multiple segments and choosing proper Lyapunov – Krasovskii functions, two delay-dependent criteria for absolute stability of systems in infinite sector and finite sector were respectively derived. The derived stability criteria were both discrete delay dependent and neutral delay dependent. The proposed condition was in terms of a linear matrix inequality which can be easily solved with Matlab toolbox. Finally, numerical example demonstrates the validity and feasibility of the proposed criteria.

Key words: Lurie control system; delay dependent stability; neutral delay; linear matrix inequality

(上接第 39 页)

Set Pair Analysis Model for Bridge Landscape Assessment

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Abstract: In view of uncertainty and complexity in the process of bridge landscape assessment, based on set pair analysis (SPA) theory, identity – discrepancy – contrary (IDC) connection degree formula which can embody certainties and uncertainties of the assessment system is introduced and set pair analysis model (SPAM) is established in bridge landscape assessment. The meaning of SPA applied to bridge landscape assessment in two levels is fully analyzed. All indexes can be summarized in five – member connection number. Analytic hierarchy process (AHP) is applied to calculate the weighing values in order that the artistic degree of the bridge can be calculated from the objective and subjective evaluations. As an example, Hulu river bridge is assessed with SPAM. The results show that: from the whole system, the artistic degree of the bridge is fine; from the subsystem, bridge esthetics, modeling and environment coordination are better than others. The results express the artistic degree of the bridge on different levels and different aspects. SPAM combines qualitative description with quantitative analysis and takes full advantage of uncertainty. It is a new model for the bridge landscape assessment.

Key words: bridge engineering; landscape assessment; set pair analysis